Developing a Framework to Understand Mathematics at the Teacher/ Learner Interface.

Geoff Wake and Birgit Pepin

University of Manchester[1], UK Høgskolen i Sør-Trøndelag, Norway

This paper explores the construct of narrative as a key mediating instrument within a Cultural Historical Activity Theory (CHAT) analysis of mathematics classrooms and uses this to consider how what counts as 'engaging students in mathematics' may be constructed very differently from setting to setting. As a starting point we consider classrooms situated in social systems that are organised through regularities of shared practice (Gesalfi et al 2009; Bauersfeld 1992; Yackel & Cobb 1996; Cobb 2000), and we consider that these systems influence the ways that individuals are expected, entitled, and given opportunities to participate (Gee 1999; Holland et al 1998), taking account for example of culturally established 'rules' and responsibilities, and 'divisions of labour'. Here, we focus on a specific aspect of the classroom activity system - the ways in which teachers mediate learning through their 'mathematical stories' suggesting that each teacher has a particular 'story', a particular way of controlling student participation, leading to a particular process of 'negotiating' (or directing) what it means to 'do mathematics' and become (or not) a mathematician.

Key words: pedagogy, narrative, cultural historical activity theory.

Background and theoretical frame

The Economic and Social Research Council (ESRC) funded research project, 'Keeping open the door to mathematically demanding courses in Further and Higher Education' involved both case study research investigating classroom cultures and pedagogic practice, and individual students' narratives of identity. This qualitative strand was complemented by quantitative analyses of measures of value added to learning outcomes in an attempt to investigate the effectiveness of two different programmes of AS mathematics for post-16 students. In this paper for illustrative purposes we focus on experiences in just one classroom from one of five case study colleges. We draw on data that was collected in the ethnographic tradition: video and audio recordings; photographs; researcher notes; follow-up interviews with students both in small groups and individually; and pre- and post- lesson interviews with the teacher involved.

---

[1] Now at University of Nottingham, U.K. (geoffrey.wake@nottingahm.ac.uk)

We conceptualise the classroom interactions we observed across our five case study centres as nested within an evolving systems network, in which teacher and students are mutually constituted through the course of such interactions. The notion of close relationship between social processes and developing knowledge draws on the work of post-Vygotskian activity theorists (e.g. Cole, Engström, Holland etc), and is fundamental to Lave and Wenger's (1991) social practice theory which emphasises the notions of "community of practice" and collective knowledge that may emerge within the spaces people share and within which they participate.

We found a wide range of practices employed by teachers as they attempted to engage their students with mathematics, with the resulting activities being more or less teacher- or student- centred. For example, these included:

- monological transmission of information by the teacher to the whole class;
- alternating phases of group-discussion followed by whole-class discussion;
- students "playing a game" (with the teacher facilitating this by setting the rules and time-frame).
-

These different pedagogic practices appear dominant in setting a 'tone' for the classrooms of different teachers, and they clearly impact in a major way on students (see for example, Hernandez-Martinez et al, this issue). It is our contention that understanding lessons as *pedagogic events* and consequently students' experiences requires careful analysis if we are to better understand their potential impact on students' learning as well the likelihood of their continued engagement in its study.

It is clear that the instruments that teachers employ, including discourse (Wertsch, 1991) are essential in mediating the mathematics at issue: however, we propose that not only are 'signs' and language crucial in the development of learning, but also of importance are the teacher's careful construction and unfolding of their mathematical argument. We therefore consider the teacher as storyteller: the teacher weaving together episodes in the development of her particular mathematical argument, and with each episode contributing to a significant plot reflecting her epistemological and pedagogical knowledge and beliefs that helps to determine the cultural models of mathematics afforded by her lessons, and thus with which students are expected to engage. We therefore explore the mathematical development of lessons through the lens of "narrative", in the sense of Ricouer (1984), and as developed in educational settings by Bruner (1996) and others. This positions the teacher as "narrator", revealing a mathematical plot whilst drawing on a range of pedagogic practices in an attempt to engage his or her audience in different ways.

In justification of our claim that teachers use narrative constructions in relation to mathematics in their classrooms we state here briefly some of the key features of narrative in general, as identified by Bruner (1996), before we exemplify this thinking in relation to a particular lesson:

1) Sequencing of events: the unfolding of crucial events in a temporal sequence, with a beginning (a problem to be solved), middle and end (resolution of this).
2) Hermeneutic composition: how the episodes of a narrative have meaning on their own, but how in combination they provide greater meaning than their individual parts and, in retrospect, how the whole narrative adds meaning to the individual constituent episodes.
3) "Trouble": narratives in general should run counter to expectancy and have "trouble" as a central feature.

These key features are, we suggest, identifiable in the way that teachers unfold mathematics for their students: thus we are led to propose that mathematics itself is developed as a distinct narrative by teachers in their classrooms with the plot devised by the teacher reflecting.their knowledge in relation to subject and pedagogy (Shulman 1986; Ball et al 2008) Alongside this developing mathematical narrative we also note the social interaction between students and teacher and the narrative that the teacher develops in relation to student engagement. From a semiotic point-of-view, and as Morgan (2006, p.220-21) points out,

> "Every instance of mathematical communication is thus conceived to involve not only signification of mathematical concepts and relationships but also interpersonal meanings, attitudes and beliefs. … Individuals do not speak or write simply to externalise their personal understandings but to achieve effects in their social world."

Thus, all 'utterances' relate to both the mathematics as well as to directing behaviour: in our case the story to 'model' the mathematics, as well as affording pupil engagement, where both reflect the teacher's perceptions of how this can be achieved. Furthermore, in a narrative individuals *position* themselves in particular ways, and this refers to the ways people use action and speech to arrange social structures (Harré & Langenhove 1999). The teacher's choice of words and/or associated actions are likely to evoke images of 'known stories' and positions within those stories. For example, the teacher may position herself as a 'researcher' measuring and exploring 'worm growth'; yet another as a 'coach' and the students as 'motivated athletes'. Any utterance in these kinds of classroom conversations are likely to cast participants in certain roles in a known 'storyline' (discourse) (Wagner & Herbel-Eisenmann 2008).

Crucial, therefore, are the discursive social interactions between teachers and students: we detect that whilst at times these may be spontaneous they are also often more deliberate than that and planned to interweave and interplay with the developing mathematical narrative in such a way that the social narrative may add power to, and strengthen it.

Thus, in an attempt to analyse lessons as pedagogic events we are led to using a two-dimensional framework (Figure 1) with one dimension taking into account the teacher's narrative about the mathematics itself; and a second dimension which is socially focused and takes account of the pedagogic practices that the teacher uses and which reflects the culture of mathematics teaching and learning within which the teacher works. It is our contention, therefore, that a student's experience lies at the intersection of these two dimensions with each relying on a discourse as well as actions / ways of working that signal what it means to learn, understand and do mathematics as well as become a (type of) mathematician. Therefore, even if two teachers were to have the same mathematical narrative, it is unlikely that their social narrative and choice of pedagogic practices would be the same, and consequently students in their two classes would have different experiences, even before the crucial issue of their interpretation of these experiences is taken into account.

Figure 1 about here

Interpreting the narrative of a lesson

We now turn to a lesson which we recount in some detail, identifying in the unfolding episodes characteristics of this particular teacher's story, based on the framework proposed above.

This lesson is in the first term of the AS "Pure" course in a college in the North of England. It focuses on applications of differentiation with the teacher in an interview some time before the lesson observation telling us:

> "My teaching style has always been quite traditional. Pretty much a typical lesson of mine will be aim and objectives on the board or recap what we did last time, an introduction of what we're doing this time and I expect students to make notes because I don't just… literally only use the textbook for questions, I often teach a topic differently than in the textbook and then students try some…"

This summary is borne out by the lesson transcripts that give stark evidence of a monological transmission style with very few utterances by anyone other than the teacher. In the introductory phase of the lesson the teacher initially drew attention to differentiation as the abstract idea of rate of change of y with respect to x referring to the notation $\frac{\mathrm{d}y}{\mathrm{d}x}$. Perhaps the transmission style the teacher employed is encapsulated in his statement at this point of the lesson that, "we just need a couple of definitions before we can move on to what I wanted to look at in detail today."

He drew a non-specific/general curved line and emphasised that the gradient at a specific point is given by the differential of the function introducing the appropriate notation $f(x)$ and $f'(x)$. At this point he introduced the "new stuff" – the average gradient, or gradient of the chord, between two points (*A* and *B*) on the curve, although he did suggest that this idea had been met by the group when differentiation was first introduced. Again the transmission and teacher-centric style employed is evident at this particular point as he stated that, "what I'm about to say now are the two most important things you need to learn this lesson." Here he re-emphasised that the gradient at a point is found using differentiation, $\frac{dy}{dx}$, and the average gradient between two points is found using the gradient of a chord, as he introduced the notation $\frac{\delta y}{\delta x}$.

In a second phase of the lesson the teacher modelled how to answer a problem of a type that they would practise in the final stage of the lesson. However, at this point of the lesson the teacher introduced a 'social' strand of narrative that from this point interweaves with the mathematical narrative. This revolved around an imagined world in which the teacher developed a problem situation about the worms in his garden: this is not a 'real' context but perhaps is 'realisable' (e.g. Van den Heuvel-Panhuizen 2001). As this extract of the transcript of the lesson demonstrates this strand of the teacher's narrative with which he expects the students to engage is not insubstantial:

> "So I went into my garden, true story this, and I started digging up some worms. Alright? So I took my fork and I dug up lots of worms and they were all of different sizes so that's quite interesting in the first place. So I thought, well, I wonder if there's any relation between the age of these worms and their length so I collected as many worms as I had time for. For the visual learners amongst you, here's one of them. This is, in fact, it's Japanese. Could be German. Who knows? Ok, this is one of the worms I collected. So I collected till I'd got enough, a decent sample size, right? And I measured these worms, how long they were. I then asked them how old they were. They were quite co-operative. And I plotted how long the worms were at particular ages and, to my surprise, and remember you're not making notes, this is background, to my surprise, when I plotted the age of the worm to its length, all the points roughly lay on what looks to me like a quadratic so, of course, as you yourselves, I got quite excited at that and I thought, well, if I could find the equation of that quadratic, I'm quids in, yeah? I could predict the length of worms at different ages that I didn't have so I got very excited. I

also noticed that when the worm wasn't born its age was zero so that was spot on, that fits nicely, so I do know one point that lies on this potential quadratic, quadratic with a negative coefficient of the squared term."

With brief reference to techniques that students had met previously of fitting a quadratic curve to model data such as this, the teacher went on to introduce the equation $l = 8t - \frac{1}{2}t^2$ that he claimed to have found for a curve that fits his imaginary data of worm length, *l* millimeters, at time, *t* years. The first part of the problem that he set was to find the rate of growth of these worms in the first year of their lives. He immediately translated this applied problem for the students into the more abstract mathematical form of having to calculate "delta *l*" by "delta *t*". He then proceeded to demonstrate how to find the average gradient by firstly finding *l* when $t = 1$ and then proceeding to calculate the increase in *l* $\left(7\frac{1}{2} - 0\right)$ divided by the increase in *t* (ie 1). After brief comment by the teacher that a rate of growth of $7\frac{1}{2}$ millimetres per year was, "Quite a lot really" he repeated the procedure carrying out all of the calculations at each stage to find the average rate of growth during the fourth year. In conclusion of this phase of the lesson the teacher "discussed", by asking questions that he answered, the validity of the answers he had found so far:

> "Has the result surprised you or not? 7.5 millimeters per year in the first year, 4.5 millimeters per year in year 3 to year 4. Does that make sense that a worm grows really quickly at first and then starts slowing down its growth rate? That seems sensible to me, I think we do the same. Obviously, I'm still growing but…ok."

In what may, due to the shift in the mathematics involved, be considered a third phase of the lesson the teacher posed the question,

> "What is the rate of growth after 3 years? Not what is the average rate of growth. Exactly, on the worm's third birthday - at that instant, what is its rate of growth?"

Again in this phase the teacher modelled how to find an answer by differentiating the function $l = 8t - \frac{1}{2}t^2$ and substituting $t = 3$ to give a rate of change of 5 millimetres per year with the class asked to consider the likely validity of this answer by comparing it with the average rates of change he had found for the first and third years of the worm's life.

In the next phase of the lesson the teacher posed the question, "How many years before the worm is fully grown?" After suggesting that the students should think about this in terms of the rate of change of the length of the worm a student made the second intervention of the lesson suggesting that this is at a stationary point. Re-interpreting this, the teacher pointed out,

> "In other words, the gradient is zero. When the worm is now fully grown, it's no longer growing so the rate of change of the length with respect to time is zero."

Due to restricted space here we leave any further description of this lesson, which continued much in this vein, other than drawing attention to a penultimate phase in which students practised the techniques introduced and recapped in the lesson during which, on the whole, they worked individually.

Analysis: narrative strands in the activity system of the 'worms' classroom

This particular lesson had relatively little variation in pedagogic practice, with the teacher in the main choosing to use a predominant transmission style for long stretches with the only change to this being a substantial phase near the end of the lesson in which students practised the techniques that he had modelled to solve a range of similar problems. This resulted in a long period of passive activity for the students followed by a period in which they were more actively engaged but on the whole working individually: the result was little or no sociability throughout the lesson.

It is the different strands of the teachers' narrative, be they mathematical or social, that we suggest lie at the heart of the learners' experiences of mathematics, defining and delimiting the lesson's distinctive phases or episodes. In our analyses of lessons different phases are often discernable due to a change of pedagogic practice employed by the teacher, and therefore by the different modes of engagement of teacher and students. For example, in the penultimate phase of this lesson the students practise their application of techniques whereas in the final phase they hear the "trailer" for the forthcoming lesson. However, at other times different phases are suggested by a distinctive shift in mathematics: for example, early in this lesson students have to shift their thinking from considering how to find average gradient to considering the behaviour of a quadratic function. We suggest, therefore, that different phases of lessons can be identified by reference to changes in pedagogic practices or introduction of a new chapter in either mathematical or 'social' narrative.

In this particular lesson the teacher demonstrates the use of the two distinct strands of mathematical and "social" narrative and interweaves these, at times using the 'social' narrative to motivate and at other times ensuring it intersects relatively closely with the mathematics in such a way that engagement with the social requires engagement with the mathematical. For example, consider how the teacher's social and mathematical narratives are closely aligned as he discusses how to find when a worm is fully grown with students thinking about growth mathematically (considering the maximum point of the quadratic function) and socially (the teacher emphasizes that in this context the function would suggest that the worms would be shrinking and that "we can't have that for worms".)

We consider the mathematical narrative of this lesson to be of the genre "abstract skills and techniques have use in solving contextual problems" with the teacher's narrative comprising of a number of episodes that in the main build on previously met skills, techniques and understanding. These episodes introduce:

- how to find the average gradient between two points on the graph of a function (strongly emphasized as a technique to be practised);
- the idea that a quadratic function can be used to model "real" data;

- how to find the gradient at a point using differential calculus when a function is known;
- the idea that the validity of calculations and indeed the choice of model should be interpreted and considered in the light of the context under consideration.

Here, the teacher's 'social' narrative focuses almost exclusively on his imagined growth of the worms in his garden. He uses this, at times with a touch of humour, as general incentive for engagement but at other times to introduce the mathematical processes of interpretation and validation of the model introduced for the growth of the worms (consider, for example, how he suggests that his worms should not be shrinking after eight years as the model might suggest). We suggest that the stories the teacher spins about his worms require more attention than just focusing on the 'social': the mathematical is at times intricately interwoven with this. Equally, sharing the worm story is also a central element of this teacher's dual goal-directed action (engagement with, as well as learning of, mathematics): he wants students to connect to the worm story in order to learn about the mathematics. This requires particular action, and knowledge of stories that allow him to develop the mathematics. It also requires actions in terms of clarifying, establishing and enforcing particular 'participation' (or non-participation) rules within the classroom community.

An activity-theoretic perspective

So far we have considered the mediating influence of the teacher's narrative on the student's engagement with mathematics in isolation from other key factors. We now turn to consider how these interact from an activity theory perspective. The well-known schema (Engestrom 1987), Figure 2, draws our attention to key mediating aspects of an activity system such as that of the classroom. In considering our unit of analysis, as the classroom activity system, our attention is drawn to the interaction of the community of teacher and students and their division of labour. The stark division of labour we observe in 'the worms' classroom with the active teacher and passive students, together with the teacher's transmissionist practices are clearly reflected in the teacher's narrative form.

Figure 2 about here

We note, for example, that this particular teacher put great emphasis on the 'I' (*teacher*) rather than the 'we' (*students and teacher*), indicating clearly the division of labour. Pimm (1987) and Rowlands (e.g. 1992) point to the strong interpersonal role personal pronouns (e.g. 'we' or 'I') can play in such situations. Our teacher also mentions in interview that the 'rules' of his classroom include that students have to be absolutely quiet, and that he expects 'no talking', when he talks. Student discourse and peer discussion does not seem to be part of his pedagogic practice. The style appears directed and 'didactic': he imparts knowledge to the students. He is a confident teacher, experienced and valued in his department and school, and he says that his personal style is appreciated by his students. Thus, for him, students mainly have to listen during his lessons, occasionally answering the odd directed and closed question– these were his 'rules' of engagement that reflect his chosen 'division of labour'.

We contrast this classroom which we found to have characteristics positioned closer to the 'norm' of our observations than another in which the teacher positioned herself and her students as jointly working as a community with 'sociable' classroom practices that engage students in co-constructing understanding. In this other teacher's classroom we observed a very different division of labour (Fig. 3) in which teacher and students work as a community of inquiry (for example Yackel and Cobb 1996) with, however, the teacher very clearly directing or orchestrating activity in a way that kept close control of her developing mathematical narrative (Wake & Pampaka 2008).

Figure 3 about here

This teacher employed a distinctly different narrative form in both social and mathematical strands with constant talking, amongst students as well as students and the teacher. Whole class discussion was interspersed by student group discussions with the emphasis being on the 'we':

> "OK, we continue to look at what we do with data. We're going to look at a couple of diagrams. We are going to look at data collection and see what we can do with that. …"

'Real data' previously produced by the students during a phase of pedagogic practice that saw students involved in a whole-class experiment were used to engage students in the mathematics such that this teacher's pedagogic practices can be characterised by discourse processes and taking communication about mathematics as a central focus. We noted later in the lesson how discovery learning processes seemed to produce some student uncertainties about what to do next and how to go about it; yet the teacher confidently 'orchestrated' a purposeful learning environment where clear directions in terms of mathematical development could be identified.

This teacher's actions focused on whole class participation and active engagement in dialogue. To accomplish this depended to a large extent on a shared understanding of the importance of dialogue and the sharing of mathematical ideas. For this teacher effective pedagogy demanded careful listening to student's articulation of ideas, and subsequently acting upon these, in terms of orchestrating the mathematical discourse, guiding student thinking and practice. Jaworski (2004) provided evidence of teachers such as this noticing and subsequently acting 'knowledgeably' as they interacted at critical moments in the classroom when students created a moment of choice or opportunity. In this environment it seems vital to develop a sensibility for redirecting the discussion to ensure that important mathematical ideas are being developed, and this may be dependent on a range of pedagogical content knowledge, particularly in relation to content knowledge and students (Ballet al 2008). In this particular case we found the teacher to be highly connectionist (Askew et al, 1997; Swan, 2006) and with strong sense and understanding of her students individual needs and progress.

Conclusion

Our CHAT analysis of mathematics lessons as pedagogic events suggests that as teachers operationalise their actions in an attempt to reach their goal of engaging students with the learning of mathematics they develop different narrative constructions that can be considered as an interweaving of two different strands:

i. a mathematical strand, which includes the development of a mathematical argument, and signals a certain positioning of mathematics (epistemologically and in relation to mathematical practices)
ii. a social strand which comprises of social activities (which arise from the teacher's choice of pedagogic practices) and social discourse.

The mathematical strand is driven by the mathematical argument that the teacher wants to present and reflects the way in which the teacher understands how mathematical ideas and processes familiar to his or her students may be (re-) introduced and interconnected to develop new (to the students) mathematics. On the other hand, the social strand contains references to 'why' as the teacher draws on a range of practices and discourse with which he or she attempts to motivate and engage his or her students in learning.

In terms of Bruner's key features of narratives, we suggest that the teacher as narrator makes similar choices about the building blocks of the mathematics they seek to introduce: for example, as in all narrative forms, they too use narrative devices such as flash-forwards, perhaps using a glimpse of the end-point to act as an advance-organiser or motivator for their class. Moreover, in terms of hermeneutic composition this appears to be an important feature of mathematical narrative where, in coming to understand a particular concept or develop fluency with a particular mathematical procedure, one needs to draw on concepts, ideas and procedures met earlier and on occasions across different branches of mathematics. This feature of mathematical narrative we suggest, may be the essence of "connectionist teaching" (e.g. Askew et al 1997, Swan 2006) as the teacher weaves a richly textured mathematical landscape with many connections and links both within and to outside of mathematics. "Trouble" in mathematical narrative seems particularly pertinent when the narrator wants to engage the audience in a dialogic pedagogy. As Ryan & Williams (2007) point out such a feature, a 'problematic', is essential when the teacher wants to provoke different points of view from a shared understanding of an initial situation. Presumably the fact that, in mathematics classrooms, teachers are often confronted with common misconceptions, their narratives, as interpreted by their students, are often responsible for introducing elements that run contrary to expectations and even, for at least some, surprise. Indeed, many mathematics educators suggest that teachers should plan an 'element of surprise' into their lessons (for example see, Movshovitz-Hadar 1988).

Here, in the case of only one teacher, due to space restrictions, we have elaborated how social and mathematical strands demonstrate the key features of narrative as the teacher engages their class in the development of a unique revelation of new mathematics. The 'social' narrative in particular, we suggest, is extremely important in engaging the learners not only through different activities but also a discourse that can both afford and influence learning and which can also ensure, to a greater or lesser extent, connectivity with the mathematics itself. This appears potentially important as students attempt at the time, and later during periods of reflection, perhaps as they practise newly learnt techniques and so on, to make sense of the place of this new mathematics in the grand scheme (or wider narrative) of the discipline. Indeed, on occasions we have observed that the audience (in this case the students), become part of the narrative themselves: the social strand of narrative ensures they are fully incorporated into the development of the mathematics itself. Importantly this is planned by the teacher from the outset, who although having key episodes in the mathematical development that she wishes to ensure are 'revealed', is willing to be flexible in this regard to ensure that the students themselves co-construct the narrative of the lesson. This contrasts, on the other hand, with the lesson described here, where the involvement of the students with the narrative is less active: although the teacher introduces a relatively prominent social strand, relating to his worms, he does not incorporate pedagogic practices that might ensure his students are engaged with this. Consequently, we argue, they may have some difficulty in becoming actively engaged with the mathematical strand of the teacher's narrative and such narrative interaction certainly does not support discover or connectionist practices.

These 'stories' (and events) we consider are nested within a system network which means that the contingency of student engagement and understanding through a teacher's narratives depends on a network of interrelated factors and environments. Whilst all case study teachers were said to be 'effective' (by their colleagues and students), their 'stories' were very different, and in turn afforded different kinds of engagement. Whilst 'effectiveness' is often taken to be almost entirely measurable by assessment outcomes as our project highlighted students' dispositions towards mathematics and in turn the mathematicians they become is contingent on their experiences in and with classroom narratives.

References

Askew, M., M. Brown, V. Rhodes, D. Wiliam and D. Johnson, D. 1997. *Effective Teachers of Numeracy.* London: King's College.

Ball D. L., M. H.Thames and G. Phelps. 2008. Content Knowledge for Teaching: What Makes It Special? *Journal of Teacher Education*. 59: 389.

Bauersfeld, H. 1992. Classroom cultures from a social constructivist's perspective. *Educational Studies in Mathematics,* 23: 467-81.

Bruner, J. 1996. *The Culture of Education*. Cambridge MA.: Harvard University Press.

Cobb, P. 2000. Accounting for mathematical development in the social context of the classroom. In *Radical constructivism in action: Beyond the pioneering work of Ernst von Glaserfeld*, ed. L.P.Steffe& P.Thompson, 152-178. London: Falmer.

Engström, Y. 1999. Innovative learning in work teams: analyzing cycles of knowledge creation in practice. In *Perspectives on activity theory*, ed. Y. Engström, R.Miettinen & R.Punamäki, pp.377-404. Cambridge, UK: Cambridge University Press.

Gee, J.P. 1999. *An introduction to discourse analysis: Theory and method.* London: Routledge.

Gesalfi, M. , T. Martin, V. Hand and J. Greeno 2009. Constructing competence: an analysis of student participation in the activity systems of mathematics classrooms. *Educational Studies in Mathematics*, 70: 49-70.

Harré, R. and L. van Langenhove. 1999. *Positioning theory: Moral contexts of intentional action.* Oxford: Blackwell.

Holland,D., D. Skinner, W. Lachicotte Jr, and C.. 1998 *Identity in cultural worlds.* Cambridge, MA: Harvard University Press.

Jaworski, B. 2004. *Investigating mathematics teaching*. London: Falmer.

Lave,J and E. Wenger. 1991. *Situated learning: legitimate peripheral participation.* New York: Cambridge University Press.

Morgan, C. 2006. What does social semiotics have to offer mathematics education research, *Educational Studies in Mathematics,* 61: 219-45.

Moshovitz-Hadar, N. 1988. School Mathematics Theorems – an endless source of surprise. *For the Learning of Mathematics,* 8, 3:.34-40.

Pimm, D. 1987. *Speaking mathematically.* New York: Routledge and Kegan Paul.

Ricoeur, P. (1984) *Time and Narrative*, Volume 1. Translated by Kathleen McLaughlin and David Pellauer.

Rowlands, T. 1992. Pointing with pronouns. *For the Learning of Mathematics,* 12 (2): 44-48.

Ryan, J., and J. Williams. 2007. *Children's Mathematics 4-15. Learning From Errors and Misconceptions*. Maidenhead, Open University Press.

Shulman, L. S. 1986 Those Who Understand: Knowledge Growth in Teaching. *Educational Researcher*, 15 (2): 4 – 14.

Swan, M. (2006) *Collaborative Learning in Mathematics – A Challenge To Our Beliefs and Practices.* London, NRDC.

Van den Heuvel-Panhuizen, M. 2001. Realistic Mathematics Education as Work in Progress. In *Common Sense in Mathematics Education. Proceedings of The Netherlands and Taiwan Conference on Mathematics Education,* ed. F.L. Lin. Taipei: Taiwan.

Wagner, D. and B. Herbel-Eisenmann B. 2008. "Just don't": the suppression and invitation of dialogue in the mathematics classroom. *Educational Studies in Mathematics,* 67: 143-57.

Wake, G. and M. Pampaka. 2008. The central role of the teacher - even in student centred pedagogies. In, Proceedings of the Joint Meeting of PME 32 and PMENA XXX In *Proceedings of the Joint Meeting of the 32nd Conference of the International Group for the Psychology of Mathematics Education and the XX North American Chapter.* Ed. O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano, & A. Sepulveda. Vol. 4, 377–384. Morelia, Mexico: Cinvestav-UMSNH.

Walshaw, M. and G. Anthony, G. 2008. The teacher's role in classroom discourse: a review of recent research into mathematics classrooms. *Review of Educational Research*, 78 (3): 516-51.

Wertsch, J.V. 1991. *Voices of the Mind: A Sociocultural Approach to Mediated Action.* London: Harvester.

Wood,T. 2002. What does it mean to teach mathematics differently? In *Mathematics education in the South Pacific Proceedings of the 25th annual conference of the Mathematics Education Research Group of Australasia,* ed. B.Barton, K.Irwin, M.Pfannkuch & M.Thomas, Vol.1, 61-67. Sydney: MERGA.

Yackel, E., and P. Cobb. 1996. Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education,* 27: 458-77.

Figure 1

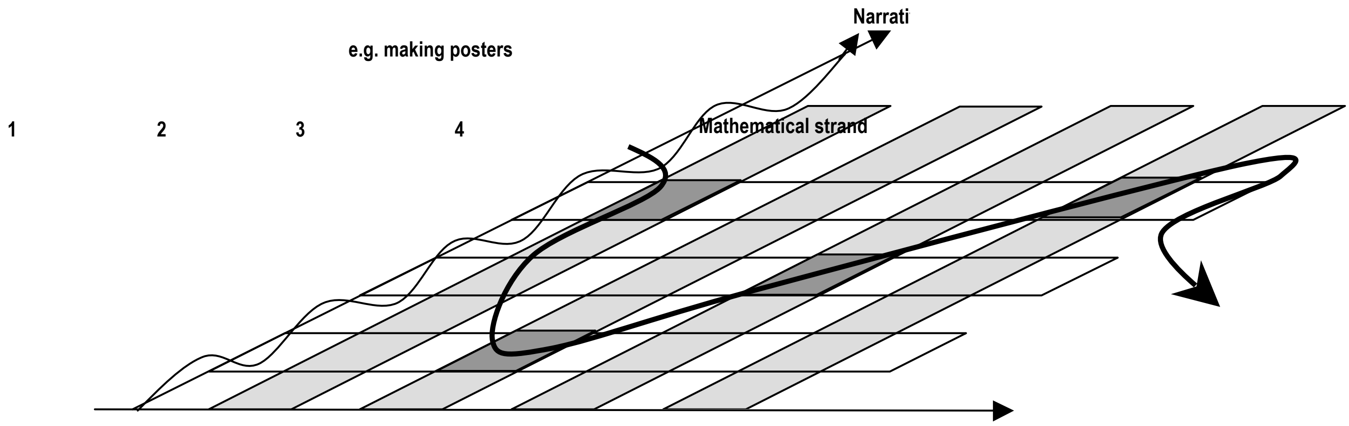

Figure 1. Schema illustrating two dimensional framework used to analyse the narrative of mathematics lessons

Figure 2

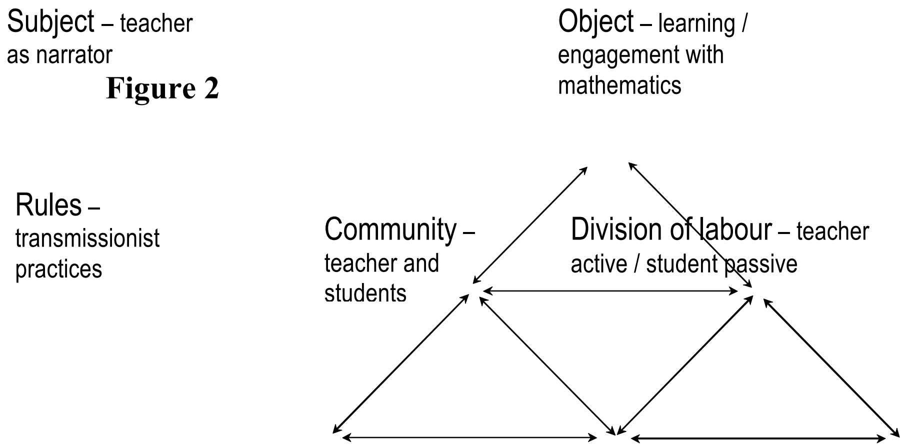

Figure 2. Activity system: participation in the ‘worm story’

Subject – teacher as narrator

Rules – connectionist practices

Instruments – narrative

teacher and students

Object – learning / engagement with mathematics

and students both active but teacher central orchestrator

Figure 3

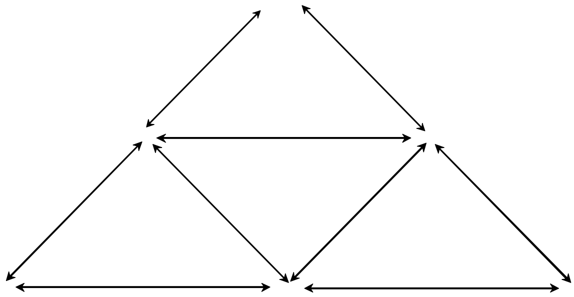

Figure 3. Activity system of participation in an ‘inquiry classroom’.