Developing Higher Level Skills in Mathematical Modelling and Problem Solving Project

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Abstract

In this project mathematical modelling skills were introduced and assessed for students on undergraduate engineering programmes related to the built and natural environment through a first year mathematics module. A substantial number of students on the module were work-based and had not studied mathematics for some time. To support this work-based cohort, the delivery incorporated a blend of class contact and distance learning support through web-based materials and computer based assessments. We were therefore able to compare the performance of a traditional class-based delivery mostly involving students straight from school and the work-based students who were taking the module as part of their continuing professional development. As might be expected, the latter group were the stronger performers as evidenced by their end of module performance, although the initial evaluation did not suggest a substantial difference between the two groups in terms of their awareness of mathematical modelling or the use of mathematics in an applied context. The project has produced a series of formative and summative computer based assessments that can be delivered on the Blackboard virtual learning environment (VLE) to support the module delivery. As a second strand of activity, an outreach event involving mathematical modelling and problem solving was delivered to year 12 A-level mathematics students in collaboration with Mathematics for Education and Industry towards the end of the project. Initial evaluation of this event indicated a high degree of satisfaction from both students and teachers.

Background and Rationale:

The project was initiated to improve engagement of engineering students with their mathematical studies and develop higher level transferable skills from the study of mathematics. The UWE strand of this project is one of five such strands, the others being delivered at the Universities of Leeds, Manchester and Keele, involving students on undergraduate programmes in mathematics, physics and engineering. Mathematical modelling and problem solving are important transferable skills that all high level numerate undergraduate programmes expect to develop. However, the experience of our own institution and the observations of others, reported regularly at the IMA Engineering Mathematics conferences and HE-STEM/MSOR meetings, is that many students struggle to relate the mathematics they have learnt at school and at University to the solution of engineering problems. The difficulties that students experience in formulating a mathematical model to describe a physical problem was the subject of a study [1] which identified the reduced exposure to mechanics and Newton's laws of motion of A-level students in both mathematics and physics as being a contributory factor in this problem. Pre-2000, it may have been a reasonable expectation that students entering university courses in mathematics, engineering and physics would be familiar with the language and some of the stages involved in mathematical modelling. However, this can no longer assumed to be the case.

The role of the mathematics module used in the project has been and is to provide students from a variety of academic backgrounds (in mathematics) with the mathematical techniques required by other modules in their programme. In its delivery and assessment we have balanced the requirement that the mathematical technique of the students is developed further as well as developing mathematical modelling skills. The assessment of the new module was designed to reflect this requirement while providing the stimulus for students to apply their mathematical knowledge. The module was assessed by coursework weighted at 40% of the final mark and an end of module examination weighted at 60%. The coursework component consisted of four equally weighted elements comprising of three written assignments which concentrated on applications of mathematics and modelling and a fourth element that comprised a series of four computer based tests assessing a mixture of mathematical technique and applications. The end of module examination had two equally weighted sections, one on mathematical technique, the other on applications and modelling.

Describing the background of the students on the module in more detail, we have two cohorts;

- The first cohort consists of students on the Civil Engineering, Building Services engineering, Architecture and Environmental Engineering programmes. There were 65 students in this cohort. Students on these programmes will possess either an A-level in mathematics, a B-TEC level 3 qualification, Access to Higher Education qualification or will have passed the Engineering Foundation Year at UWE. These students study the course through three hours contact per week over twenty four weeks. The contact takes the form of a two hour whole cohort session which will typically involve lecture material and problem solving classes and a one hour workshop.

- The second cohort consists of students on the River and Coastal Engineering programme, currently working for the Environment Agency or Local Authorities in flood risk management. There were 60 students in this cohort. Many of these students will already possess a first degree but in a discipline without a strong mathematical element such as a degree in Geography. Two thirds of these students had an A-level in mathematics. The remaining third had successfully completed a pre-entry distance learning module in the preceding year to raise their mathematics from GCSE standard to AS/A2 level.

The development in the curriculum or delivery of the mathematics module must accommodate part-time students. Part-time students study our programmes in a number of different patterns either through weekly day-release whereby they study alongside the full-time students or through distance learning blended with block week study at the University, two or three times per semester. So the development of materials suitable for distance learning was an essential part of our strand of the project.

The outreach element of the programme was designed to illustrate to both students and teachers the higher level skills develop through programmes in mathematics and engineering. Through working with MEI, ourselves and the other partners will be able to share best practice and also develop outreach material that can be disseminated to schools and colleges an existing national network, The Further Mathematics Support Network. The UWE event was designed in consultation with two local HE colleges, City of Bristol college and Filton college as part of our strategy for widening access to our programmes.

Implementation:

The development and delivery of mathematical modelling skills in this project was implemented through separate deliveries to the two cohorts of students. All students had access to the same learning materials and took the same assessments. Evaluation of the project concentrated on assessing the students' initial awareness of mathematical modelling and applications, delivered through a questionnaire and a short modelling exercise, followed by collecting samples of student work from the written assessments. The evaluation is described in the next section and was implemented according to the format agreed between the four University partners, co-ordinated by the University of Manchester.

Central to delivering the aims of the project, was to ensure that the distance learning students were adequately support and were provided with sufficient feedback as to their progress. This aim was achieved through

- Use of the Blackboard VLE to provide a structured delivery of the course material based around a course text "Mathematics for Engineers" by Croft and Davison [2] supported by supplementary material from Help Engineers Learn Mathematics (HELM) [3] and exerts from "An Introduction to Mathematics for Engineers (Mechanics)" by Lee [4] and material written by the course team.
- Computer based tests that provide instant feedback
- Three half day sessions which concentrated on modelling and applications
- Access to web-based support materials through the Blackboard VLE
- Written assignments that were submitted and marked electronically, and returned to students within three to four weeks of submission.
- Email tutoring to provide regular support as required

The UWE strand of the project involved a partnership between UWE and MEI with Stephen Lee from MEI developing a support web-site using MEI's materials for A-level Further Mathematics. MEI also provided as series of scenarios for the application of mathematics and modelling which could be given to students as paper based exercises. These exercises are subsequently being converted into a computer based test format which will be added to the resources produced by the project and disseminated to project partners.

The choice of the Blackboard platform to deliver the computer based tests was chosen to assist the ease with which students could monitor their progress and to ensure that the questions could be shared with other institutions using the Blackboard VLE. Those institutions using other VLEs can take the designs and adapt the questions for their system. Questions currently developed are suitable for testing knowledge of mathematical techniques, dimensions, Newton's laws of motion and forces, applications of integration and differential equations. Building on our experience this year, further developments are planned for questions on projectiles, various water flow scenarios to added to the computer based resources produced by the project and trialled with students next year.

Due to the nature of the academic background of the students, it was decided that during this first year of delivery, to assess the students on their ability to show modelling skills such as problem formulation, use of diagrams, assumptions, interpretation of results and apply certain stages of the modelling cycle but not expect them to reach the stage where they could confidently implement a complete iteration of the cycle. For example, students have solved problems based on applying Newton's laws of motion to determine the motion of connected bodies and consider the effect of friction on the motion. While students formulated a model, solved the resulting equations and were asked to reflect on the results, they were working within the framework provided by Newton's laws motion. Whether, in their view, they were engaged in a modelling exercise or following a well defined method of solution is open to question and of course will vary from one student to another. Informed by our observations of how students coped with the material this year, the problems will be developed further to include more open ended elements.

The three written assignments were on Newton's Laws of Motion and Forces, where a problem of involving or neglecting wind resistance on the motion of a cyclist was consider, together with a problem finding the maximum number of people that could be carried by a lift given certain constraints about the acceleration of the lift and the strength of the supporting cable. The second written assignment involved an inverse square law problem of finding the minimum point of illumination on a sports pitch. This problem was structured for students to explore how to start with a simplified version of the problem and then to then modify the problem in steps to increase how realistically the model would capture the features of the problem to be solved. This problem in particular would be a strong candidate to be presented in a more open ended format in subsequent years and allowing greater scope for students to come up with an evaluate models that would clearly be incorrect. The final written assignment was on modelling the surface area of the Guerkin building in London given some partial data relating to the dimensions of the building and different forms of the a possible model for the profile of the building. Again, a more open ended form of this problem could be presented in subsequent years.

The outreach event, involving twenty two students was delivered at UWE in collaboration with MEI, the actual event was delivered with Stella Dudzic and Charlie Stripp from MEI. The planning for the event involved an initial meeting with representatives from City of Bristol College, Filton College and John Cabot Academy. The structure of the event involved a morning of problem solving and mathematical modelling activities. The afternoon involved the students undertaking a modelling and experiment activity to investigate Newton's law of cooling. The event concluded with a whole group session where we explored Newton's laws of motion and their application to the design of the Bloodhound car attempting to break the land speed record. Feedback was very positive from both teachers and students and next year it is planned to increase the number of participants and to carry out further work on the activities.

Evaluation:

As stated earlier, the evaluation of the project followed a methodology agreed between the project partners. A questionnaire was used as an initial instrument for assessing the student's awareness of mathematical modelling. This was followed by an initial modelling activity involving a short problem which was not reliant on prior specialist knowledge. Finally, samples of work illustrating the student's performance on modelling activities undertaken during the project was collected and distributed to project partners at a seminar held towards the end of the project.

We were able to obtain further information using end of module results and feedback from the end of module questionnaires and feedback sessions

In the module population there were 65 students in the class contact cohort and 60 students in the distance learning cohort.

	Questionnaire	Short Modelling Exercise
Class contact cohort (65)	25	4
Distance Learning cohort (60)	57	33

Table 1: Engagement of cohorts in evaluation exercise

The engagement of the distance learning cohort in the initial evaluation exercises was far greater than the class contact cohort. However, from the initial questionnaire we were able to gain some insight into the initial disposition of the students to skills related to mathematical modelling.

The questions asked, are given below for reference, with students required to rank their confidence on a five point scale from very negative to very positive. They could also record a "Don't know response.

1	Solving practical, real-life problems with mathematics.
2	Reading and interpreting mathematics in a practical task
3	Learning some new mathematics for use in your work
4	Using mathematics to solve puzzles for leisure
5	Working in a team where you have to listen to others explain some mathematics
6	Working in a team where you have to explain some mathematics to others
7	Devising a mathematical representation for a situation in order to solve a problem
8	Drawing a diagram to model a situation when solving a problem
9	Defining variables and parameters that might be important in a problem
10	Researching a situation which demands understanding some mathematics
11	Writing a report of how you used mathematics for a non-specialist audience

Table 2: Initial Questionnaire

The tables below show the responses of the students, with the results summarised

	Positive (%)	Neutral (%)	Negative (%)	Don't Know (%)
Q1	88	8	4	0
Q2	80	8	8	4
Q3	80	8	8	4
Q4	56	32	8	4
Q5	68	20	8	4
Q6	48	44	4	4
Q7	72	24	0	4
Q8	88	8	0	4
Q9	56	32	12	0
Q10	64	28	8	0
Q11	12	4	44	4

Table 3a: Response of class contact group

	Positive (%)	Neutral (%)	Negative (%)	Don't Know (%)
Q1	79	17	2	2
Q2	81	14	3	2
Q3	91	9	0	0
Q4	66	22	12	0
Q5	69	22	9	0
Q6	45	29	26	0
Q7	52	24	22	2
Q8	62	22	14	2
Q9	62	22	14	2
Q10	57	26	17	0
Q11	24	31	41	3

Table 3b: Response of distance learning group

The only question where both groups gave a mostly negative response was the final question on communicating mathematics to a non-specialist audience. Interestingly, the work-based student seemed less confident about working in a team on mathematical activities. On questions 7 to 9 which were about the process of modelling, the class contact group appear more confident than the distance learning group. On the other questions which relate to associated skills, attributes or disposition that would be associated with mathematical modelling there is little difference between the two groups.

In the actual short modelling activity involving a runner and swimmer racing against each other across a pool and around the perimeter of a pool, the students demonstrated some knowledge of the language and steps of developing a mathematical modelling, but clearly lacked confidence bringing these steps together to build a model, even though the exercise was highly structured with hints as to how to proceed. This was interesting given a reasonably positive response to questions 7 to 9. Very few students were able to draw an effective diagram. For an interesting comparison, a similar exercise was given to A-level maths students attending the UWE-MEI outreach event. Those students taking A-level further mathematics were very effective at setting up and solving the problem.

The work submitted by the students for the written assignments from both cohorts contained some excellent work. The understanding of the modelling process and relating the mathematics to the applications were strongest in the distance learning cohort as was the communication of the mathematics. The table below shows the end of module results for the two cohorts

	Written assignment 1	Written assignment 1	Written assignment 1	Computer based tests	Examination
Class contact	84	58	59	62	51
Distance Learning	84	70	77	63	61

Table 4: Average marks on assessed work

The above table shows that the more mature distance learners produced a much stronger performance the class contact group, the main difference being on the two longer written assignments and the end of module examination. The pass rate for the distance learning cohort was 87% whereas the pass rate for the class contact cohort was 69%. We would expect after the referred examination for the class contact pass rate to rise to close to 80%.

Feedback from both groups indicated that they found the module difficult and that the amount work involved was high compared to other modules. However, as expected given the above results, the distance learning cohort felt more confident with their mathematics and had appreciated exercises which placed the mathematics in a physics or engineering context.

Discussion, Learning and Impact (Success):

Our experience in the project during the year has confirmed the lack of confidence and experience students have in applying their mathematics to physical problems. The students on the module have come from a variety of mathematical backgrounds and no particularly background provided an advantage in going beyond mastering techniques to the application of those techniques. However, the project has shown that it is possible to engage students in developing higher level skills through mathematics. Both cohorts of students on the module found this a challenging year. Despite their initial responses in the evaluation questionnaire, it was quite clear that very few had experience of applying mathematics let alone building a mathematical model. This justified our approach in setting the modelling activities in a highly structured format.

The performance and attitude of the work-based distance learning students was excellent and the measures taken to support this group were successful. The project has enabled us to develop resources for the support of the distance learning students which have been shown to be effective. The performance of the class contact cohort was disappointing and the challenge posed by the more applied questions seems to have resulted in a small number of students disengaging from the learning process.

One issue we have identified during this project has concerned the use of a mathematics module to develop modelling skills and an appreciated of mathematical models in an engineering context. While the more mature group of work-based students engaged in the process, it was always likely that the work-based cohort were going to be highly motivated as the course is part of their continuing professional development. However, they also had the capacity to be highly critical if they did not perceive the course as meeting their needs. In fact they have been very complementary about the course and the learning support they received. Unfortunately, some of the class-based students did not seem to make the connection of the relevance of the exercises to their wider studies.

During the year we were required to embark on a process of curriculum review, replacing programme structures based on 20 credit modules with a new structure based on 30-credit modules. Consequently we have been able to introduce a new 30 credit module Mathematics and Engineering Principles to replace the module delivered in this project. The new module will develop mathematical modelling skills in a module where the applied physics content of the engineering programme is also covered. This new module should provide a more coherent vehicle for developing mathematical modelling within an engineering context and specifically allow for integrated mathematics and engineering assessment. Therefore, an impact of the HE STEM project has been to enable us to react to this curriculum refresh in a positive manner with the aims of the project of developing higher level skills informing the design of the new module. Our hope is that the new module will prove to be more effective vehicle for engaging the full-time students.

The outreach event was a very successful day with highly positive initial feedback from the participants. We are still awaiting a more detailed analysis of the evaluation feedback forms, but we will repeat and expand the exercise next year. A very positive outcome from the HE STEM project has been the way it has led to networking opportunities, sharing of experiences and resources and the development of what should be sustainable working relationships.

Further Development and Sustainability:

The work carried out in this project will be developed further and sustained in the following ways

- Materials developed in this project will be used on the 30 credit module Mathematics and Engineering Principles which will combine the development of mathematical techniques, modelling and engineering principles in dynamics and stress analysis. In this new module we intend to go further in promoting the modelling process than was possible in the current module and will be able to set integrated mathematics and engineering assessment.

- The computer based assessments developed for delivery on the Blackboard platform will be expanded and made available to project partners.

- Next year, students from the Environment Agency will be attending classes full-time at the University. However, the distance learning structure will be used to support part-time routes in Civil Engineering and improve the learning support for all students.

- The pilot outreach event on modelling and problem solving with MEI will be continued and expanded for next year. We intend to work with MEI to further develop suitable exercises that can be disseminated more widely.

References

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