The University of Manchester

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TransMaths Research Briefing

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Learning to connect with, use, and apply mathematics: mathematical modelling for problem solving in STEM

Summary findings

For many students mathematics at school or college and mathematics at university play very different roles. At university, mathematics is often studied as part of a course in engineering, the physical sciences or, increasingly, the social sciences. This means that students are required not only to learn mathematical skills and techniques but also how to apply, to model and to solve problems with mathematics. This provides students with a range of significant, substantial new challenges. However, we found that this often goes unrecognised in programme design and by lecturers in their teaching at university.

1 In pre-university maths education, we found some connectionist teaching that provided students with opportunities to work together to develop deeper understanding, and which allowed them to develop more positive relationships with, and dispositions towards, mathematics. The Use of Mathematics programme, through its coursework assessment, obliged students to engage at length with applications and mathematical modelling. This allowed students time and focussed teaching on learning, thus deepening understanding of mathematical concepts. Only rarely did we find instances of such mathematics tasks or courses in the university sector, but again in such cases we found students responding positively to such longer, often group-focussed tasks.

2 Transfer of mathematical knowledge, skills and understanding by students to situations in science, technology or engineering is frequently problematic and is not recognised as such by those responsible for university course design. This runs counter to the research literature in mathematics education which suggests that for some, learning may be so contextually situated that transfer is extremely difficult. In general in universities, we found an emphasis on teaching and learning of mathematical skills and techniques with little attention paid to issues of 'transfer' or knowledge transformation. We found that lecturers were not always sensitive to the obstacles that appears relatively 'simple' mathematics can present to students' in new contexts. This can prove problematic for students trying to making sense of subjects that assume mathematics, often at the core of their programme in STEM.

3 Application of mathematics in solving problems often requires deeper understanding of mathematical ideas than competence with 'basic' techniques and procedures. Such technical competence is necessary but not sufficient. Students need more space and time on their courses to develop deeper understanding (even of topics they may have met before). We found that support centres were often used by students who were seeking such understanding. However, not all students recognise the need for such understanding and they may not prioritise their learning accordingly. Such students need more help with their self-assessment.

Mathematics learning, identity and educational practice: the transition into Higher Education

The Transmaths research projects investigated students' transitions from school through college into mathematically demanding degree programmes in Higher Education. The focus was on transitional practices and the projects investigated the effects on learner identities, choices and learning outcomes. Using a mixed methods approach, quantitative survey data were analysed alongside a longitudinal series of student interviews and case study data.

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Summary Findings

Connectionist teaching, modelling and applications

When researching students on two different mathematics programmes in AS/A-level in preparation for university we found that those following Use of Mathematics courses developed different 'cultural models' of mathematics from those who followed the traditional A-level. In general, the students on Use of Mathematics courses, in comparison with those on traditional courses, were more likely:

- To develop an awareness that there are diverse ways of doing mathematics (although they themselves when solving problems often used only a single approach or method);
- To be aware of uses and applications of mathematics, and appreciate how mathematics can be used to model different situations;
- To talk of coming to deeper understandings of mathematics due to the 'space' that the production of coursework afforded them to engage with substantial mathematical modelling problems in meaningful contexts;
- To see mathematics as having immediate or potential 'use value' rather than purely talking in terms of its 'exchange value' as a qualification.

We found the requirement in the Use of Mathematics programme that students engage with substantial modelling tasks to produce coursework for assessment particularly important in this regard. Some students also found the use of technology beneficial, whether as a tool in learning mathematics, or as a mathematical tool used in applying mathematics . The programme, by design, affords students a wider range of different ways of engaging with mathematics than was often the case in courses leading to the traditional A-level.

Overall, our analysis of student interviews found that students had positive experiences of modelling through coursework. They appreciated how this provided them with opportunities to develop conceptual understanding and how assessment of this allowed them to gain credit in ways that traditional timed written examinations (which some students find particularly threatening) do not.

Int: So in general how did you find the coursework?
E: It wasn't so difficult. I enjoyed doing it.
Int: You enjoyed it.
E: Yeah. I learned a lot because of it.
Int: Could you tell me which parts of this assignment did you enjoy more?
E: Fitting the model. It's good when the model is perfect, or almost perfect. To see that your parameters are very good so... yeah.

AS Use of Maths and BTec Engineering student

In universities we found mathematics courses relied on lectures plus support classes for problem sheets which gave students opportunities to practise the knowledge and skills presented in lectures. Alternative programmes that engaged students with substantial problem solving tasks were rarely found, but when they were these appeared to provide motivation and engagement for students.

Our research highlights how connectionist teaching can support students in developing more positive dispositions towards study of mathematics and more positive levels of confidence and self-efficacy in learning and using mathematics than transmissionist teaching. However, as Figure 1 shows, transmissionist teaching practices dominate teaching at university: although the situation is arguably only a little better pre-university. In general connectionist teaching is sensitive to learners' prior understanding and developing needs and allows students to develop agency in their learning of, and use of, mathematics. It requires teachers to be sensitive to the emerging needs of both individuals and groups of learners and to support learning as a community activity.

The HE research project developed measures of connectionist and transmissionist pedagogies which confirm most university teaching of mathematics as transmissionist. Figure 1 highlights students' responses to questions about specific teaching practices considered as either connectionist or transmissionist.

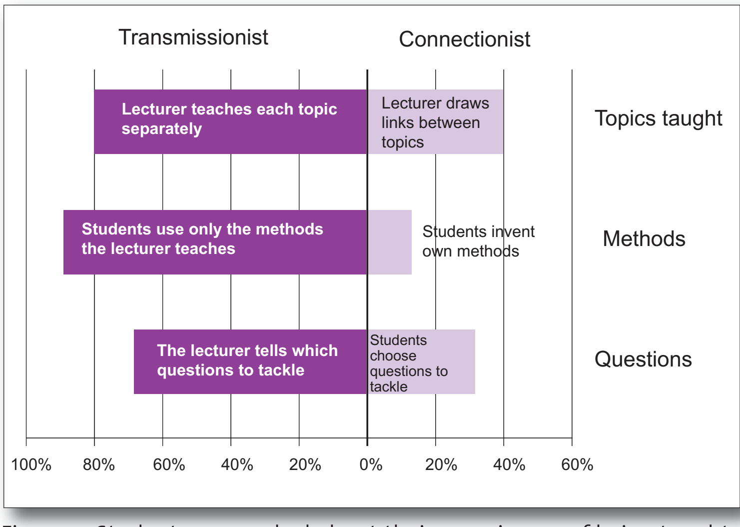

Figure 1. Students were asked about their experiences of being taught mathematics mid-way through their first year at university.

Transfer/transformation of mathematical knowledge

For many students the transition to university sees the role of mathematics change compared to their previous experiences at school or college. Once at university, mathematics for those studying mathematically demanding subjects as opposed to mathematics itself, is additionally to be applied as well as being an object of study. However, the issue of transfer or perhaps more properly, transformation, of mathematical knowledge is not acknowledged in programme design. There appears to be an assumption that competency with key mathematics techniques is both necessary and sufficient as preparation for the use of mathematics elsewhere with mathematics lectures seeming disconnected from the main areas of students' studies. Some limited sensitivity to the transfer issue is often seen in some mathematics lecturers' notes, and more widely available materials, where examples of applications are added at the end of the maths problem sheet, serving to indicate to students where they might come to apply what they have learnt

Int: And I noticed that at some points you tried to connect with the engineering part of the mathematics, is that an important aspect of the lecture?

J: It is because obviously the whole point of this mathematics course is to give them a solid foundation for their other engineering modules so that's why you know interspersed each week there should be a particular engineering example so that they can hopefully see how the maths they have just learnt can link in with something they may come across this year or next semester or next year so that's something that's brought in; that's been more of a challenge for me because I'm a mathematician I have to say I'm not an engineer so some of these examples I have to talk through and try and sound as though I know exactly what I'm talking about which is difficult because if you're teaching a maths course if you're an engineer then you've got the engineering background so you can talk absolutely about the different examples but if you're a mathematician you understand the maths behind it but not necessarily au fait with the engineering.

A-level teacher working at a university lecturing on a first year engineering mathematics course

This may be due to the fact that the 'mathematics lecturer' only knows the mathematics; but it may also be that the scientist or engineer simply does not recognise the centrality of the mathematics to the understanding of the science or engineering concepts at issue.

Of course, lecturers presenting mathematical techniques and procedures in large lectures can lead to apparent efficiencies of scale in teaching. But when students from across different science, technology, and engineering programmes are taught mathematics together, their interest in any examples used becomes problematic. In such cases we observed even less concern being given to modelling, problem solving and applications.

Our observations of lectures in engineering and physical sciences, for example, suggest that lecturers in these subjects do not recognise the mathematical demand that their exposition might require of students. For example, it may be that lecturers refer to concepts with which they are familiar and which rely on mathematical representations that have become part of their daily practice, but which cause students to stop in their tracks. In an engineering lecture we observed, for example, a lecturer referred to the simple, to him, concept of a uniformly distributed load, and how shearing force and bending force diagrams might represent this. Although these are straightforward ideas, at the first time of meeting they require students to think carefully about the mathematics underlying these standard representations.

Our student interviews reflect the educational research literature that points to transfer as being problematic for some. In particular students talk about how they have to work hard to connect an abstract world of mathematics with the reality that it models: this requires care in understanding both the mathematics and the physical situation, and for these understandings to connect. Students need time and space in their courses to engage with this connection-making.

Int: Ok, now erm, talking about maths how does maths fit into your plans?

S: Erm, to be honest I'm not too fond of maths but I do understand that it's very closely related to physics so I practise it a lot and just I always try to understand what the equations are trying to tell you and so that way I can always deduce the answers logically

Int: And what is it that you don't like about maths?

S: That almost you can't apply it it's all, you just get numbers out there's no meaning to those numbers in physics when you use the maths you get meaning to those numbers you get a feel for is the answer correct and that's what I like

Student in early first year at university studying Physics

M: So far it's been really hard. I dunno, it's hard to sort of place- whenever we're integrating in electro-magnetic fields, because I think integration has been taught so abstractly.

Int: Yeah.

M: It's hard to do, when you've actually got a sphere.

Int: Yeah.

M: You know, it's like not just like area under a graph which is what integration was- like that's all it was, the area under a graph. And if that, you know we were just literally taught to integrate, and we weren't taught any applications.

Student in second semester studying Engineering at university

In some cases mathematics plays a much more important role than is obvious to students choosing to study a subject. For example, we found students using support centres for help with mathematics in pursuit of their study of subjects such as psychology. In such a subject statistical and probabilistic models are often used in literature which students may be expected to read or even critique. In the most extreme case we became aware of a student discontinuing her studies because of mathematical demands for which she was totally unprepared.

1 Mathematics learning, identity and educational practice: the transition into post-compulsory education (ESRC grant RES-000-22-2890)
2 Keeping open the door to mathematically demanding courses in Further and Higher Education (ESRC grant RES-139-25-0241)

Implications and recommendations

Current models of teaching and learning mathematics at university, both as a subject in its own right and in support of other subjects, are narrowly restricted: usually consisting of lectures (efficient in terms of economies of scale) supported by 'problem classes' (allowing practice of techniques, rules and procedures). Almost all lecturing (and even tutoring in problem classes) is transmissionist in nature and so often does not connect with students' needs and prior experiences. The needs of students might be better supported by

- consideration being given in programme design and use of resources to allow students more time and space to develop connected and deeper understanding of mathematical concepts (for example, through working on mathematical modelling problems)
- attention being given to the issue of transfer and transformation of mathematical knowledge with the issue being recognised and addressed both within mathematics course units and other units that rely on mathematics
- lecturers of STEM subjects other than mathematics itself, showing greater sensitivity about the level of demand of the mathematics and so the difficulty posed for students
- providing more opportunities for students to engage with conceptual understanding and to develop a clearer understanding of mathematical structure and its interconnections.

Our research into the learning of mathematics at A Level suggests that programmes can be designed so that they afford opportunities for students to engage with substantial problem solving and modelling tasks that allow the development of deeper understanding of mathematics. We recommend that such approaches be further developed pre-university and in university STEM programmes.

Where next?

During 2011 the team, with ESRC Follow-on funding, will ?? work with key partners, including the National HE STEM Programme, the National STEM Centre and the National centre for Excellence in Teaching mathematics (NCETRM) to promote participation and engagement in post-compulsory mathematics education for STEM. This work will draw on and synthesise findings across all three research projects that investigated students' trajectories in and through mathematics programmes from compulsory school, through college to Higher Education. Further details of our ongoing work can be found at the project website. Additionally the team are involved with further research that builds on previous work. The ERSC funded project (grant RES-061-25-0538) 'Mathematics teaching and learning in secondary schools: the impact of pedagogical practices on important learning outcomes' will explore issues of teaching and learning in the secondary years of compulsory school.

Further information

The TransMaths projects have developed, and are continuing to work on, a range of publications and other dissemination resources that can be found at the project website: www.transmaths.org

Of particular relevance to the issues raised in this Research Briefing are:

Pampaka, M., Williams, J. and Hutcheson, G. (under review) Students experience of transition into mathematically demanding courses in university and its association with outcomes. British Educational Research Journal.

Wake, G. (2011). Introduction to the Special Issue: Deepening engagement in mathematics in pre-university education, Research in Mathematics Education

Wake, G. (under review) Making sense of and with mathematics. Special issue on modelling, Educational Studies in Mathematics.

Williams, J. (under review) The use of value in mathematics education, Educational Studies in Mathematics, special issue on contemporary theory in mathematics.

The warrant

One of the strengths of our project that investigated transitions into Higher Education is that it drew on a variety of methods. It used large-scale questionnaire surveys of students (n>1700) at the start of their university course and after the first semester, developed case studies of 13 university courses (mostly in STEM), and tracked a number of students in more depth through three longitudinal interviews (N>50). This provided a rich base of data for analysis.

In addition, we extended the conceptual framework already developed for our previous research project ESRC TLRP 'Keeping Open the Door to Mathematically Demanding Programmes in Further and Higher Education' which explored transition through college in much the same way.

Specially constructed instruments were developed and validated to measure important new affective learning outcomes in the transition into mathematically demanding (STEM) programmes in Higher Education.

The case studies were developed from mainly qualitative investigations and involved observations of lectures and tutorials with interviews of students and teachers. Triangulation was supported by the collection of other university degree course documents and data, and interviews with other stakeholders such as Heads of Departments. The series of longitudinal biographical style interviews about students' transitional experiences provided further deep description and insight into the transitional process.

Our methodological approach is imbued with the notion of generating practical knowledge in partnership with students and university teachers as informed and knowledgeable participants. This partnership approach also provides an ethical (and triangulating) basis for all the empirical, analytical and reporting work. A series of meetings with university teachers assisted in this respect. Finally, our warrant is also enriched by the project's advisory group, which consisted of academics and practitioners with relevant experience, and which met regularly with the project team.

Project website:

www.education.manchester.ac.uk/research/centres/lta/ltaresearch/transmaths/into-he/

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